Comparative Analysis of Two Optimization Models for Prediction of Compressive Strength of Lateritic Block

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Abstract – Two different optimization models for prediction of compressive strength of lateritic blocks was performed using statistical analysis for the lateritic block data obtained from experimental work done in this study. The models used are Scheffee's and Osadebe's optimization theories to predict the compressive strength of lateritic blocks. The results of predictions were comparatively analysed using the statistical package for social sciences (SPSS) for the student's t-test. It was found that the two models are acceptable for the prediction of compressive strength of lateritic blocks.

Index Terms— Compressive strength, Lateritic block, Mix proportion, Osadebe's theory, Scheffee's theory

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1 INTRODUCTION

Lateritic soils are the product of intensive weathering that occurs under tropical and subtropical climatic condition resulting in the accumulation of hydrated iron and aluminium oxides. In Nigeria, laterite is readily available and mostly used for construction or foundation material for roads, airfield and compacted fill in embankments due to their favourable engineering properties. The cost of river bed sand, which is popularly used for block production has been on the increase and becoming scarce in some areas. Laterite has been found to be a suitable alternative or partial replacement for river bed sand. According to [1], sand-laterite blocks were produced by replacing laterite partially with river sand using 10% to 40% replacement levels. This partial replacement is necessary to reduce total dependence on river sand in block production. According to the cost analysis conducted by [2], laterite blocks have 40% cost advantage over similar sandcrete blocks. Several optimization studies have been carried out for adequate prediction of compressive strength of blocks or concrete made of different components or admixture. This study compares Scheffee's and Osadebe's optimization theories to predict the compressive strength of lateritic blocks.

2 MATERIALS AND METHODS.

2.1 Materials.

The materials that were used in casting the lateritic blocks were water, cement and laterite. The equipment used include shovels, weighing balance, hand trowel, solid mould (six inches block mould), scoop etc.

Water: The water that was used in the production of the lateritic block was sourced from the public water supply in Niger Delta University Campus, Wilberforce Island, Bayelsa State, Nigeria.

Cement: Ordinary Portland cement was used.

Laterite: The laterite soil sample was collected from borrow pit from Ahoada, Rivers State, Nigeria. The red coloured laterite soil sample was air dried and crushed to fine granules.

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2.2 Experimental Method.

The experimental procedure includes sampling, weighing, mixing, moulding or casting of lateritic block, curing and laboratory analysis to determine the compressive strength of each sample of lateritic block. The actual mix proportions were measured by weight and used to produce lateritic blocks of size 450mm x 150mm x 225mm. A total of forty-eight lateritic block were cast according to the specified mix ratios. The blocks were cured for 28 days in an open place by sprinkling of water. In accordance to BS 2028 (1968) [3], the blocks were tested for compressive strength. The results obtained from the crushing tests were subjected to optimization analysis. .

2.3 Analytic Method

Scheffe's Regression Model

According to [4], the relationship between the actual Z and the uncoded or pseudo X component is linear and could be represented by the equation shown below:

$$[Z] = [A]\,[X] \qquad (1)$$

Where Z = Actual mix ratio
X = Pseudo mix ratio
A = Conversion factor.

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This could be explained by a triangle simplex for a three component in the mixture as shown below:

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad (2)$$

The general equation of regression for Scheffe is given as follows:

$$Y = b_o + \Sigma b_{ij} X^2 + \Sigma b_i X_i X_j + \Sigma b_{ijk} X_i X_j X_k + ----- + e \quad (3)$$

Where Y = the response

b_o = arbitrary constant

e = random error (i.e. the combine effects of variable excluded in the model)

Hence, establishing Scheffe's regression model equation with respect to number of components (three – pseudo component mix) at each point is as follow:

$$Y = b_o + b_1 + X_1 + b_2 + X_2 + b_3 + X_3 + b_{11} + X_1^2 + b_{12} + X_1 X_2 b_{13} + X_1 X_3 + b_{22} + X_2^2 + b_{23} + X_2 X_3 + b_{33} X_3^2 + e \quad (4)$$

Since sum of components at each vertex must be equal to unity i.e.

$$X_1 + X_2 + X_3 = 1 \quad (5)$$

Multiplying (4) by b_o will yield:

$$b_o X_1 + b_0 X_2 + b_0 X_3 = b_0$$

Similarly, multiplying (5) by $X_i (i = 1, 2, 3)$ gives

$$X_1^2 + X_1 X_2 + X_1 X_3 = X_1 \quad (6)$$

$$X_2 X_1 + X_2^2 + X_1 X_3 = X_2 \quad (7)$$

$$X_3 X_1 + X_3 X_2 + X_3^2 = X_3 \quad (8)$$

Rearranging (6) – (8) gives:

$$X_1^2 = X_1 + X_1 X_2 - X_1 X_3 \quad (9)$$

$$X_2^2 = X_2 + X_2 X_1 - X_2 X_3 \quad (10)$$

$$X_3^2 = X_3 + X_3 X_1 - X_3 X_2 \quad (11)$$

Substituting (9) – (11) into (4) will yields:

$$Y = b_o + X_1 + b_0 + X_2 + b_0 + b_3 + X_3 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_{11}(X_1 - X_1 X_2 - X_1 X_3) + b_{22}(X_1 - X_1 X_2 - X_1 X_3) + b_{33}(X_3 - X_1 X_3 - X_2 X_3) + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{23} X_2 X_3 \quad (12)$$

Rearranging (11) and collecting like terms together gives:

$$Y = (b_o + b_1 + b_{11}) + X_1 + (b_0 + b_1 + b_{12}) X_2 + (b_0 + b_1 + b_{33}) + X_3 + (b_{12} + b_{11} + b_{22}) X_1 X_2 + (b_{13} + b_{11} + b_{33}) X_1 X_3 + (b_{23} + b_{22} + b_{33}) X_2 X_3 \quad (13)$$

Introducing constants, we have:

$$b_o + b_1 + b_{11} = A_1 \quad (14)$$

$$b_o + b_1 + b_{22} = A_2 \quad (15)$$

$$b_o + b_1 + b_{33} = A_3 \quad (16)$$

$$b_{12} + b_{11} + b_{22} = A_{12} \quad (17)$$

$$b_{13} + b_{11} + b_{33} = A_{13} \quad (18)$$

$$b_{23} + b_{22} + b_{33} = A_{23} \quad (19)$$

Substituting (14) – (19) into (13) we have:

$$Y = A_1 X_1 + A_2 X_2 + A_3 X_3 + A_{12} X_1 X_2 + A_{13} X_1 X_3 + A_{23} X_2 X_3 \quad (20)$$

According to Scheffe, (20) could be written in the form:

$$Y = \Sigma A_i X_i + \Sigma A_{ij} X_i X_j \quad (21)$$

Recall that summation of un-coded component at any point on the simplex must be equal to one.

$\therefore n_i = A_i$ which mean

$$n_1 = A_1; \quad (22)$$

$$n_2 = A_2; \quad (23)$$

$$n_3 = A_3 \quad (24)$$

Where n = response function for pure components.

At the mid-point of the border line connecting points 1 and 2 of the factor space, component $X_1 = 1/2$ and that of $X_2 = 1/2$ while $X_3 = 0$, the response at this point is n_{12}

Similarly, substituting the mid-point conditions into (20) gives:

$$n_{12} = \frac{1}{2} A_1 + \frac{1}{2} A_2 + \frac{1}{4} A_{12} \quad (25)$$

$$n_{13} = \frac{1}{2} A_1 + \frac{1}{2} A_3 + \frac{1}{4} A_{13} \quad (26)$$

$$n_{23} = \frac{1}{2} A_2 + \frac{1}{2} A_3 + \frac{1}{4} A_{23} \quad (27)$$

In summary,

$$n_{ij} = \frac{1}{2} A_i + \frac{1}{2} A_j + \frac{1}{4} A_{ij} \quad (28)$$

Rearranging (22) and (28), we have:

$$A_i = n_i \quad (29)$$

$$A_{ij} = 4n_{ij} - 2A_i - 2A_j \quad (30)$$

Substituting (29) into (30) yields:

$$A_{ij} = 4n_{ij} - 2n_i - 2n_j \quad (31)$$

Substituting these values in equation (29) and (31) into (20) and collect like terms will give:

$$Y = n_1 X_1 + n_2 X_2 + n_3 X_3 + (4n_{12} - 2n_1 - 2n_2) X_1 X_2 + (4n_{13} - 2n_1 - 2n_3) X_1 X_3 + (4n_{23} - 2n_2 - 2n_3) X_2 X_3 \quad (32)$$

$$Y = n_1 X_1 + n_2 X_2 + n_3 X_3 + 4n_{12} X_1 X_2 - 2n_1 X_1 X_2 - 2n_2 X_1 X_2 + 4n_{13} X_1 X_3 - 2n_1 X_1 X_3 - 2n_3 X_1 X_3 + 4n_{23} X_2 X_3 - 2n_2 X_2 X_3 - 2n_3 X_2 X_3 \quad (33)$$

$$Y = (n_1 X_1 - 2n_1 X_1 X_2 - 2n_1 X_1 X_3) + (n_2 X_2 - 2n_2 X_1 X_2 - 2n_2 X_2 X_3) + (n_3 X_3 - 2n_3 X_1 X_3 - 2n_3 X_2 X_3) + 4n_{12} X_1 X_2 + 4n_{13} X_1 X_3 + 4n_{23} X_2 X_3 \quad (31c)$$

$$Y = n_1 X_1 (1 - 2X_2 - 2X_3) + n_2 X_2 (1 - 2X_1 - 2X_3) + n_3 X_3 (1 - 2X_1 - 2X_2) + 4n_{12} X_1 X_2 + 4n_{13} X_1 X_3 + 4n_{23} X_2 X_3 \quad (34)$$

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Recall (5)

$$X_1 + X_2 + X_3 = 1$$

Multiplying (5) by 2 gives:

$$2X_1 + 2X_2 + 2X_3 = 2 \quad (35)$$

Subtracting 1 from both sides of (35), we have:

$$2X_1 - 1 = 1 - 2X_2 + 2X_3 \quad (36)$$

$$2X_2 - 1 = 1 - 2X_1 + 2X_3 \quad (37)$$

$$2X_3 - 1 = 1 - 2X_1 + 2X_2 \quad (38)$$

Substituting (36) – (38) into (32) gives:

$$Y = n_1X_1(2X_1 - 1) + n_2X_2(2X_2 - 1) + n_3X_3(2X_3 - 1) + 4n_{12}X_1X_2 + 4n_{13}X_1X_3 + 4n_{23}X_2X_3 \quad (39)$$

Substituting the value of n_1, n_2 …………, $n_{2,3}$ from Table 2 into (36a), we obtained:

$$Y = 1.250X_1(2X_1 - 1) + 1.292X_2(2X_2 - 1) + 0.950X_3(2X_3 - 1) + 4(1.620)X_1X_2 + 4(0.897)X_1X_3 + 4(1.173)X_2X_3$$

$$Y = 2.5006X_1^2 - 1.250X_1 + 2.5846X_2^2X_2 - 1.292X_2 + 1.9X_3^2 - 0.950X_3 + 6.48X_1X_2 + 3.588X_1X_3 + 4.692X_2X_3 \quad (40)$$

Where Y = compressive strength, X_n (n = 1, 2 and 3) = pseudo components for water, cement and laterite respectively.

Equation (40) is the final three components Scheffe's Model equation. On substituting the pseudo mix ratio of different points of observation as used in the experiment into (40), compressive strength of laterite blocks shown in Table 4 were obtained.

Osadebe's Regression Model

Second order Osadebe's regression equation for five components is given by [5] as:

$$Y = \alpha_1Z_1 + \alpha_2Z_2 + \alpha_3Z_3 + \alpha_4Z_4 + \alpha_5Z_5 + \alpha_{12}Z_1Z_2 + \alpha_{13}Z_1Z_3 + \alpha_{14}Z_1Z_4 + \alpha_{15}Z_1Z_5 + \alpha_{23}Z_2Z_3 + \alpha_{24}Z_2Z_4 + \alpha_{25}Z_2Z_5 + \alpha_{34}Z_3Z_4 + \alpha_{35}Z_3Z_5 + \alpha_{45}Z_4Z_5 \quad (41)$$

Equation (41) is generally expressed as:

$$[Y_n] = [Z_n][\alpha_n] \quad (42)$$

Where $Y_n = experimental\ compressive\ strength,$
$Z_n = pseudo\ components,$
$\alpha_n = coefficient\ of\ the\ model\ and$
$n = the\ number\ of\ observation.$

Equation (42) can be expanded for a three component mix at six points of observation as:

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \end{pmatrix} = \begin{pmatrix} Z^1{}_1 & Z^1{}_2 & Z^1{}_3 & Z_1Z^1{}_2 & Z_2Z^1{}_3 & Z_2Z^1{}_3 \\ Z^2{}_1 & Z^2{}_2 & Z^2{}_3 & Z_1Z^2{}_2 & Z_2Z^2{}_3 & Z_2Z^2{}_3 \\ Z^3{}_1 & Z^3{}_2 & Z^3{}_3 & Z_1Z^3{}_2 & Z_2Z^3{}_3 & Z_2Z^3{}_3 \\ Z^{12}{}_1 & Z^{12}{}_2 & Z^{12}{}_3 & Z_1Z^{12}{}_2 & Z_2Z^{12}{}_3 & Z_2Z^{12}{}_3 \\ Z^{13}{}_1 & Z^{13}{}_2 & Z^{13}{}_3 & Z_1Z^{13}{}_2 & Z_2Z^{13}{}_3 & Z_2Z^{13}{}_3 \\ Z^{23}{}_1 & Z^{23}{}_2 & Z^{23}{}_3 & Z_1Z^{23}{}_2 & Z_2Z^{23}{}_3 & Z_2Z^{23}{}_3 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{pmatrix} \quad (43)$$

$$[\alpha_n] = [Z_n]^{-1}\,[Y_n] \quad (44)$$

$$Z_i = \frac{S_i}{S} \quad (45)$$

$$S = S_1 + S_2 + S_3 \quad (46)$$

$$\sum Z = 1 \quad (47)$$

A triangle simplex was used in this study to depict six points interaction for three components namely: water, cement and laterite. At the vertices, the actual mix ratios, for example are N1 (0.55: 1: 3.5); N2 (0.65: 1: 4.5) and N3 (0.75: 1: 5.5). The corresponding pseudo mix ratio are N1 (1: 0: 0), N2 (0: 1: 0) and N3 (0: 0: 1).

Point 1: Substituting the mix ratio from point N1 into equation 1 gives:

$$\begin{pmatrix} 0.55 \\ 1 \\ 3.5 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (48)$$

Solving equation above we have:

$0.55 = a_{11} \times 1 + a_{12} \times 0 + a_{13} \times 0; \therefore a_{11} = 0.55$
$1 = a_{21} \times 1 + a_{22} \times 0 + a_{23} \times 0; \therefore a_{21} = 1$
$3.5 = a_{31} \times 1 + a_{32} \times 0 + a_{33} \times 0; \therefore a_{31} = 3.5$

Point 2: Substituting the mix ratio from point N2 into (1), we have:

$$\begin{pmatrix} 0.65 \\ 1 \\ 4.5 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad (49)$$

Solving equation above we have:

$0.65 = a_{11} \times 0 + a_{12} \times 1 + a_{13} \times 0; \therefore a_{12} = 0.65$
$1 = a_{21} \times 0 + a_{22} \times 1 + a_{23} \times 0; \therefore a_{22} = 1$
$4.5 = a_{31} \times 0 + a_{32} \times 1 + a_{33} \times 0; \therefore a_{32} = 4.5$

Point 3: Substituting the mix ratio from N3 into (1) gives:

$$\begin{pmatrix} 0.75 \\ 1 \\ 5.5 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (50)$$

Solving equation above we have:

$0.75 = a_{11} \times 0 + a_{12} \times 0 + a_{13} \times 1; \therefore a_{13} = 0.75$
$1 = a_{21} \times 1 + a_{22} \times 0 + a_{23} \times 1; \therefore a_{23} = 1$
$5.5 = a_{31} \times 1 + a_{32} \times 0 + a_{33} \times 1; \therefore a_{33} = 5.5$

Substituting the elements of matrix A into (1), we have:

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \quad (51)$$

Point 1, 2

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$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 0.55 & 0.65 & 0.75 \\ 1 & 1 & 1 \\ 3.5 & 4.5 & 5.5 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.6 \\ 1 \\ 4 \end{pmatrix} \quad (52)$$

Point 2, 3

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 0.55 & 0.65 & 0.75 \\ 1 & 1 & 1 \\ 3.5 & 4.5 & 5.5 \end{pmatrix} \begin{pmatrix} 0 \\ 0.5 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 0.7 \\ 1 \\ 5 \end{pmatrix} \quad (53)$$

Point 1, 3

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 0.55 & 0.65 & 0.75 \\ 1 & 1 & 1 \\ 3.5 & 4.5 & 5.5 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 0.65 \\ 1 \\ 4.5 \end{pmatrix} \quad (54)$$

These give the corresponding actual mix ratio as N1,2 (0.6: 1: 4); N2,3(0.7: 1: 5) and N1,3(0.65: 1: 4.5). The first six mix ratio could be summarized as shown in the Table 1.

Applying (41) for each point we have:

Point N_1

$Z_1 = \frac{S_1}{S}, Z_2 = \frac{S_2}{S}, Z_3 = \frac{S_3}{S}$ from (45)

S = actual mix ratio

$S = 0.55 + 1 + 3.5 = 5.05$

$Z^1{}_1 = \frac{0.55}{5.05} = 0.11, \quad Z^1{}_2 = \frac{1}{5.05} = 0.20, \; Z^1{}_3 = \frac{3.5}{5.05} = 0.69$

$Z^1{}_1 + Z^1{}_2 + Z^1{}_3 = 1 \; (\sum Z_i = 1)$

$\therefore \quad Z_1 Z^1{}_2 = 0.11 \times 0.2 = \mathbf{0.02}$

$Z_1 Z^1{}_3 = 0.11 \times 0.69 = \mathbf{0.08}$

$Z_2 Z^1{}_3 = 0.2 \times 0.69 = \mathbf{0.14}$

Point N_2

Similarly, S = 0.65 + 1.00 + 4.50 = 6.15

$Z^2{}_1 = \frac{0.65}{6.15} = 0.11, \quad Z^2{}_2 = \frac{1.00}{6.15} = 0.16, \; Z^2{}_3 = \frac{4.5}{6.15} = 0.73$

$\therefore \quad Z_1 Z^2{}_2 = 0.11 \times 0.16 = \mathbf{0.02}$

$Z_1 Z^2{}_3 = 0.11 \times 0.73 = \mathbf{0.08}$

$Z_2 Z^2{}_3 = 0.16 \times 0.73 = \mathbf{0.12}$

Point N_3

S = 0.75 + 1 + 5.5 = 7.25

$Z^3{}_1 = \frac{0.75}{7.25} = 0.10, \quad Z^3{}_2 = \frac{1.00}{7.25} = 0.14, \; Z^3{}_3 = \frac{5.5}{7.25} = 0.76$

$\therefore \quad Z_1 Z^3{}_2 = 0.10 \times 0.14 = \mathbf{0.01}$

$Z_1 Z^3{}_3 = 0.10 \times 0.75 = \mathbf{0.08}$

$Z_2 Z^3{}_3 = 0.14 \times 0.75 = \mathbf{0.11}$

Point $N_{1,2}$

$S^{12} = 0.6 + 1 + 4.0 = 5.6$

$Z^{12}{}_1 = \frac{0.6}{5.6} = 0.11, \quad Z^{12}{}_2 = \frac{1.00}{5.6} = 0.18, \; Z^{12}{}_3 = \frac{4.0}{5.6} = 0.71$

$\therefore \quad Z_1 Z^{12}{}_2 = 0.11 \times 0.18 = \mathbf{0.02}$

$Z_1 Z^{12}{}_3 = 0.11 \times 0.71 = \mathbf{0.08}$

$Z_2 Z^{12}{}_3 = 0.18 \times 0.71 = \mathbf{0.13}$

Point $N_{1,3}$

$S^{13} = 0.65 + 1 + 4.5 = 6.15$

$Z^{13}{}_1 = \frac{0.65}{6.15} = 0.11, \quad Z^{13}{}_2 = \frac{1.00}{6.15} = 0.16, \; Z^{13}{}_3 = \frac{4.5}{6.15} = 0.73$

$\therefore \quad Z_1 Z^{13}{}_2 = 0.11 \times 0.16 = \mathbf{0.02}$

$Z_1 Z^{13}{}_3 = 0.11 \times 0.73 = \mathbf{0.08}$

$Z_2 Z^{13}{}_3 = 0.16 \times 0.73 = \mathbf{0.12}$

Point $N_{2,3}$

$S^{23} = 0.75 + 1 + 5.0 = 6.75$

$Z^{23}{}_1 = \frac{0.75}{6.75} = 0.11, \quad Z^{23}{}_2 = \frac{1.00}{6.75} = 0.15, \; Z^{23}{}_3 = \frac{5}{6.75} = 0.74$

$\therefore \quad Z_1 Z^{23}{}_2 = 0.11 \times 0.15 = \mathbf{0.02}$

$Z_1 Z^{23}{}_3 = 0.11 \times 0.74 = \mathbf{0.08}$

$Z_2 Z^{23}{}_3 = 0.15 \times 0.74 = \mathbf{0.11}$

Substituting these values into (39)

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \end{pmatrix} = \begin{pmatrix} 0.11 & 0.20 & 0.69 & 0.02 & 0.08 & 0.14 \\ 0.11 & 0.16 & 0.73 & 0.02 & 0.08 & 0.12 \\ 0.11 & 0.14 & 0.76 & 0.01 & 0.08 & 0.11 \\ 0.11 & 0.18 & 0.71 & 0.02 & 0.08 & 0.13 \\ 0.11 & 0.16 & 0.73 & 0.02 & 0.08 & 0.12 \\ 0.11 & 0.15 & 0.75 & 0.02 & 0.08 & 0.11 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{pmatrix} \quad (55)$$

From (44) we have:

$[\alpha_n] = [Z_n]^{-1} [Y_n]$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{pmatrix} = \begin{pmatrix} 0.11 & 0.20 & 0.69 & 0.02 & 0.08 & 0.14 \\ 0.11 & 0.16 & 0.73 & 0.02 & 0.08 & 0.12 \\ 0.11 & 0.14 & 0.76 & 0.01 & 0.08 & 0.11 \\ 0.11 & 0.18 & 0.71 & 0.02 & 0.08 & 0.13 \\ 0.11 & 0.16 & 0.73 & 0.02 & 0.08 & 0.12 \\ 0.11 & 0.15 & 0.75 & 0.02 & 0.08 & 0.11 \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \end{pmatrix} \quad (56)$$

The sixteen actual mix ratios could be summarized as shown in the Table 2.

Where Z_i (i = 1, 2, & 3) = pseudo component for water, cement and laterite respectively.

$S_i \; (i = 1, 2, \& \; 3) =$ actual mix for water, cement and laterite respectively.

S = Summation of the actual mix ratio for water, cement and laterite at each point.

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$[Z_n]^{-1} =$

13007550	75399200.1	25758253.15	-63234130.55	37818717.78	-88749589.58
546375.12	3023507.92	989114.556	-2598234.077	1526468.828	-3487231.348
127512.5	753992.001	262812.5002	-0626080.5005	378225.0003	-896460.5008
-888573.3	-108620088	-36842521.46	91470361.13	-54544914.69	127422895.6
-1.5710815	-91233032	-31224753.15	76444429.12	-45761442.79	107485614
-145985.9	-757761.96	-232214.5549	673036.5381	-384229.3222	847155.1733

(57)

Determination of the Coefficient of Osadebe's Model.

The coefficients, α are obtained by multiplying the inverse of the Z-matrix by the Y-vector which is the compressive strength result obtained in the laboratory for the first six mix ratios – $n_1, n_2, n_3, n_{12}, n_{13}, n_{23}$.

$$[\mathrm{Z-matrix}][\mathrm{Y-vector}] = [\mathrm{Coefficients}] \quad (58)$$

$$\begin{pmatrix} 13007550 & 75399200.1 & 25758253.15 & -63234130.55 & 37818717.78 & -88749589.58 \\ 546375.12 & 3023507.92 & 989114.556 & -2598234.077 & 1526468.823 & -3487231.348 \\ 127512.5 & 753992.001 & 262812.5002 & -626080.5005 & 378225.0003 & -896460.5008 \\ -8885733 & -108620088 & -36842521.46 & 91470361.13 & -54544914.69 & 127422895.6 \\ -15710815 & -91233032 & -31224753.15 & 76444429.12 & -45761442.79 & 107485614 \\ -145985.9 & -757761.96 & -232214.5549 & 673036.5381 & -384229.3222 & 847155.1733 \end{pmatrix} \begin{pmatrix} 1.250 \\ 1.292 \\ 0.950 \\ 1.620 \\ 0.897 \\ 1.173 \end{pmatrix}$$

$$= \begin{pmatrix} -34447104 \\ -1400348 \\ -343046.1 \\ 49739287 \\ 41665391 \\ 356992.76 \end{pmatrix} \quad (59)$$

From (40), we have:

$$Y = \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3 + \alpha_{12} Z_1 Z_2 + \alpha_{13} Z_1 Z_3 + \alpha_{23} Z_2 Z_3 \quad (60)$$

Substituting the values of the coefficient, α_n from (59) above into (60) we obtained:

$$Y = -34447104 Z_1 - 1400348 Z_2 - 343046.1 Z_3 + 49739287 Z_1 Z_2 + 41665391 Z_1 Z_3 + 356992.76 Z_2 Z_3 \quad (61)$$

Equation (61) is the final three components Osadebe's model equation. On substituting the values of Z_i of the sixteen mix ratios used in this study into (61), compressive strength of laterite blocks as shown in Table 5 were obtained.

3 Results and Discussion

3.1 Experimental results.

The experimental test data of laterite block for twenty-eight days compressive strength obtained in the laboratory are shown in Table 3.

Comparison of Scheffe's and Osadebe's Model.

The compressive strength of the laterite block obtained from the two models are shown in Table 6. It could be observed from Table 6 that the highest percentage (%) difference Scheffe's and Osadebe's predicted compressive strength of laterite block is 1.78 %. This shows that both models predict values that are very close to each other. Hence, one could say that that there is no significant difference between the values predicted by both models.

3.2 Test for Adequacy of the Models.

Adequacy test for the models

A statistical adequacy test for the mathematical models is necessary. For this the statistical hypothesis is used as follows:

i. Null hypothesis, H_o: There is no significant difference between the two models.

ii. Alternative hypothesis, H_1: There is a significant difference between the two models.

Scheffe's Model results were tested for adequacy with the experimental results obtained in the laboratory by two tailed t-test as shown in Table 7. The results of predictions were comparatively analysed for the student's t-test. The results shows that tcal = 0.476 using paired-samples t-test. At α = 0.05, df = 9, ttable = 2:26. Since, ttable > tcal It shows that there is no significant difference between the experimental result and the results predicted by the model. Since there exist no difference between the experimental test results and Scheffe's predicted results on one hand and there is no difference between Scheffe's and Osadebe's predicted results on the other hand, it could be infers that there is no difference between experimental test result and Osadebe's predicted result. The results from this study agree with results obtained in the application of Scheffe's model in optimization of compressive strength of lateritic concrete [6], [7].

4 Conclusion

The study showed that the various models produce equivalent approximate compressive strength respectively for each mix ratio. The model equations were tested for adequacy using t-test on ten (10) controlled design points which proved that the models are adequate for use. The strengths predicted by the models are in good agreement with corresponding experimentally observed results obtained in the laboratory. Therefore, using any of these model (Scheffe's and Osadebe's model), any desired compressive strength of laterite block could be easily determined given the mix proportion. Conversely, various mix proportions matching any stipulated compressive strength can easily be obtained.

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Table 1: Values of actual mix proportions and their corresponding fractional portions for a 3-component mixture.

Points	Water	Cement	Laterite	Response
1	0.55	1	3.5	n1
2	0.65	1	4.5	n2
3	0.75	1	5.5	n3
1,2	0.6	1	4	n12
1,3	0.65	1	4.5	n13
2,3	0.7	1	5	n23

Table 2. Summary of the Sixteen Actual Mix Ratios

Points	S1	S2	S3	S
n1	0.55	1	3.5	5.05
n2	0.65	1	4.49	6.14
n3	0.75	1	5.5	7.25
n12	0.60	1	3.995	5.595
n13	0.65	1	4.5	6.15
n23	0.70	1	4.995	6.695
e1	0.65	1	4.4967	6.1467
e2	0.675	1	4.7475	6.4225
e3	0.625	1	4.2475	5.8725
e4	0.65	1	4.495	6.145
e5	0.617	1	4.1667	5.7837
e6	0.633	1	4.3283	5.9613
e7	0.666	1	4.655	6.321
e8	0.683	1	4.8267	6.5097
e9	0.633	1	4.3233	5.9563

Where S = Summation of actual mix ratio for the three components (laterite, cement and water).

Table 3: Experimental Compressive Strength Test Results for 28 Days of Laterite Block.

Points	Replicate 1 (N/mm^2)	Replicate 2 (N/mm^2)	Replicate 3 (N/mm^2)	Average Compressive Strength (YE)N/mm^2
N1	1.60	1.17	1.0	1.250
N2	1.01	1.18	1.69	1.292
N3	1.17	0.67	1.01	0.950
N1,2	2.01	1.17	1.68	1.620
N1,3	1.01	1.01	0.67	0.897
N2,3	1.34	1.34	0.84	1.173
C1	1.68	1.17	1.01	1.287
C2	1.17	1.51	1.01	1.230
C3	1.01	0.67	0.67	0.783
C4	1.51	1.34	1.34	1.397
C5	1.68	1.59	1.51	1.593
C6	1.51	1.26	1.01	1.260
C7	2.01	0.84	0.67	1.173
C8	1.59	1.01	1.01	1.198
C9	1.17	1.34	1.17	1.226
C10	1.17	0.67	1.01	0.947

Table 4: Compressive Strength of Laterite Block obtained by Scheffe's Model.

Points	Mix Ratios			Compressive Strength (YM)
	Water	Cement	Laterite	

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				N/mm²
N1	0.55	1.00	3.500	1.2503
N2	0.65	1.00	4.490	1.2923
N3	0.75	1.00	5.500	0.9500
N1,2	0.60	1.00	3.995	1.6200
N1,3	0.65	1.00	4.500	0.8970
N2,3	0.70	1.00	4.995	1.1730
C1	0.64	1.00	4.460	1.2495
C2	0.68	1.00	4.750	1.1222
C3	0.63	1.00	4.250	1.2715
C4	0.65	1.00	4.500	1.3457
C5	0.62	1.00	4.170	1.3858
C6	0.63	1.00	4.330	1.1545
C7	0.68	1.00	4.830	1.2794
C8	0.68	1.00	4.830	1.1574
C9	0.63	1.00	4.330	1.4829
C10	0.69	1.00	4.920	0.9554

Table 5: Compressive Strength of Laterite Block Obtained by Osadebe's Model.

Points	Mix Ratios			Pseudo Components			Compressive Strength(YM) N/mm2
	Water	Cement	Laterite	Z1	Z2	Z3	
N1	0.55	1.00	3.500	0.108910891	0.198019802	0.69306930	[illegible]
N2	0.65	1.00	4.490	0.105863192	0.16286645	0.73127035	[illegible]
N3	0.75	1.00	5.500	0.103448276	0.137931034	0.75862069	[illegible]
N1,2	0.60	1.00	3.995	0.107238606	0.17873101	0.71403038	[illegible]
N1,3	0.65	1.00	4.500	0.105691057	0.162601626	0.73170703	[illegible]
N2,3	0.70	1.00	4.995	0.104555639	0.149365198	0.74607916	[illegible]
C1	0.64	1.00	4.460	0.105747800	0.162688923	0.73156327	[illegible]
C2	0.68	1.00	4.750	0.105099260	0.155702608	0.73919813	[illegible]
C3	0.63	1.00	4.250	0.106428267	0.170285228	0.72328650	1.2510
C4	0.65	1.00	4.500	0.105777055	0.16273393	0.731489015	1.3217
C5	0.62	1.00	4.170	0.106679115	0.172899701	0.720421184	1.3675
C6	0.63	1.00	4.330	0.106184893	0.167748645	0.726066462	1.1361
C7	0.68	1.00	4.830	0.105363075	0.158202816	0.736434109	1.2611
C8	0.68	1.00	4.830	0.10492035	0.15361691	0.74146274	1.1465
C9	0.63	1.00	4.330	0.106274029	0.167889462	0.725836509	1.4581
C10	0.69	1.00	4.920	0.104670862	0.15125847	0.744070668	0.9520

Table 6: Comparison of the Compressive Strength Results Obtained by the Two Models.

Points	Scheffe (S) N/mm²	Osadebe(O) N/mm²	Difference (S-O) N/mm²	% Difference
N1	1.2503	1.2506	-0.0003	-0.024
N2	1.2923	1.2926	-0.0003	-0.023
N3	0.9500	0.9502	-0.0002	-0.021
N1,2	1.6200	1.6203	-0.0003	-0.0185
N1,3	0.8970	0.8973	-0.0003	-0.0334
N2,3	1.1730	1.1733	-0.0003	-0.0255
C1	1.2495	1.2282	0.0213	1.705
C2	1.1222	1.1119	0.0103	0.918
C3	1.2715	1.2510	0.0205	1.612
C4	1.3457	1.3217	0.0240	1.783
C5	1.3858	1.3675	0.0183	1.321
C6	1.1545	1.1361	0.0184	1.594
C7	1.2794	1.2611	0.0183	1.430
C8	1.1574	1.1465	0.0109	0.942
C9	1.4829	1.4581	0.0248	1.672
C10	0.9554	0.9520	0.0034	0.356

Table 7. Compressive block strength from laboratory versus Predicted Compressive block strength (Scheffe's Model).

Points	YE	YM	DI (YE – YM)
C1	1.287	1.25	0.037
C2	1.230	1.12	0.11
C3	0.783	1.272	-0.489
C4	1.397	1.346	0.051
C5	1.593	1.386	0.207
C6	1.260	1.155	0.105
C7	1.173	1.279	0.106
C8	1.198	1.157	0.041
C9	1.226	1.483	0.257
C10	0.947	0.955	0.008